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SRI LANKAN ELEMENTARY SCHOOL CHILDREN'S ERRORS IN MULTIPLICATION

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Abstract

Children's achievements in mathematics is not at satisfactory level in elementary and secondary schools in Sri Lanka. Children's mathematical errors, is one of the main reason for this low achievements. The main objective of this study is to identify the error-patterns made in multiplication by the elementary school children. Apaper-pencil test was administered to 980 children from 24 schools in Colombo District, studying in Grades 4 and 5. Each paper consisted of 50 questions. The answer scripts were marked and the responses were analyzed. 300 children were selected randomly from the 980 children and interviewed, for an in-depth study in order to identify theerror-patterns. The errors identified were classified as Careless Errors, Random Errors and Systematic Errors. Most of the errors identified were Systematic Errors. The study categorized the Systematic Errors (56.21%), further, as errors in detail: Recalling Error(9.11%),Operation Sign related Error(17.66%), Zero related Error(11.76%),Direction of Operation Error(12.46%),Answer in Reverse Order (5.36), Errors related to Carrying Digit related Errors (28.03) and Place Value related Error (15.4%). This study suggests that, teachers have to enhancetheir teaching methodologies when they teach mathematics. They have to use the relevant teaching-learning materials to explain the basic concepts of multiplications.

Key Words: Careless error, random error, systematic error, multiplication errors, Place value related errors

Introduction

Mathematics is the science that deals with the logic of shape, quantity and arrangement. Mathematics is the foundation for everything in the daily lives of people, including mobile devices, architecture art, money, engineering, and even sports.

Primary mathematics consist of six main topics – Numbers, Mathematical Operations, Measurement, Money, Space and Shapes and Data Handling. These concepts are introducedat Grade One and developed through up to Grade Five in the primary cycle (Mukunthan, 2013). The four Mathematical Operations - Addition, Subtraction, Multiplication and Division are included under the main topic Mathematical Operations.All four mathematical operations Addition, Subtraction, Multiplication and Division are introduced at Primary level.

Multiplication gives the result of combining groups of equal sizes.Whole-number multiplication is one of the fundamental skills that children acquire in primary school mathematics.The idea of multiplication starts from Grade 3. Multiplication of numbers not exceeding two digits by 2, 3, 4, 5 and 10, without carrying over is in Grade 3 syllabus, Multiplication of numbers not exceeding two digits by numbers from 2 to 10 is in Grade 4 and Consolidating multiplication of two digit numbers by numbers from 2 to 10 is in Grade 5.

Low performance in Mathematics is evident not only at the primary level but also at the secondary level. Many Sri Lankan children make errors on this concept for several reasons. Therefore, this study attempts to find the causes for these errors in multiplication.

Literature Review

An error could mean different things in different contexts. It could occurdue to several reasons. It could be the result of carelessness, misinterpretation of symbols, lack of relevant experience or knowledge related to that mathematical topic/learning objective/concept, a lack of awareness or inability to check answer given or the result of a misconception (Drews, 2005). Errors can be defined as something a person has done which is reflected to be incorrect. A mathematical error may be described as a mistake in a calculation. In the classroom, students make a variety of mathematical errors.

In mathematics, there are lots of misconceptions. These misconceptions can appear at any time during a child's education. Some misconceptions take place due to the nature of the child; other misconceptions are the results of the teaching techniques used.

According to Mulhern and Greer (1989)

a) *Errors are frequently 'surprising' usually because they may have remained undetected by teacher for some time*
b) *Errors are often extremely 'persistent'. Typically, these are resistant to change themselves and correction of errors may involve fundamental reorganization of pupil's knowledge.*
c) *Frequently errors 'ignore meaning'. So that an answer which is obviously incorrect is rejected with no questioning.*

Errors can occur in many ways, as described above. According to Brousseau et al (in Mulhern and Greer, 1989) errors may occur in four main ways in mathematics.

a) *As a result of major misconceptions about fundamental aspects of mathematics*
b) *As a result of correct and faithful application of systematically flawed procedure*
c) *When the flawed rules and misconceptions the pupils possess are not recognized by the teacher.*
d) *Due to the use of highly original, non-formal methods of solving problems invented by pupils.*

Mathematical errors are significant in practice because they can be used to enhance teaching and learning. Fiori and Zuccherri (2005) found that certain error patterns depend on different ethnic-cultural situations, teaching methods and algorithm used, and to what extent.

Many primary mathematics experts analyzed several types of mathematical errors. They have classified errors under few patterns. Cox (1975) classified the mathematical errors into three major categories viz:

(i) ***Careless error**: A student misses one or two problems out of five problems of a given type*
(ii) ***Random error**: A student misses three or more problems out of five problems of a given type, but no pattern is apparent*
(iii)***Systematic error**: A student misses three or more problems out of five problems of a given type, using the same incorrect process as evidenced by the presence of a repeated pattern*

Nesher (1987), used the term misconceptions to describe systematic errors without reference to a theoretical position.

According to Yang (2014), not all errors can be attributed to reasoning faults; some are simply careless errors. Error analysis is concerned with the pervasive errors (or 'bugs') which learners make, based on their lack of conceptual or procedural understanding (Ketterlin Geller & Yovanoff 2009).Error analysis, also referred to as error-pattern-analysis, is the study of errors in learners' work with a view to finding explanations for these reasoning errors (Herholdt & Sapire 2014). Error analysis is a process of reviewing the errors with an objective to provide feedback and remediation instructions to improve the learning and performance (Muthukrishnan et al 2019)

Radatz (1979) classified the errors in terms of

(1) Language difficulties. Mathematics is like a "foreign language" for students who need to know and understand mathematical concepts, symbols, and vocabulary. Misunderstanding the semantics of mathematics language may cause students' errors at the beginning of problem solving;
(2) Difficulties in processing iconic and visual representation of mathematical knowledge;
(3) Deficiency in requisite skills, facts, and concepts; for example, students may forget or be unable to recall related information in solving problems;
(4) Incorrect associations or rigidity; that is, negative transfer caused by decoding and encoding information; and
(5) Application of irrelevant rules or strategies.

Department for Education and Employment (1999) of United Kingdom published a model for diagnosing children's error.

(i) *Computational error / Careless mistake*
(ii) *Misconceptions*
(iii) *Wrong Operation*
(iv) *Over-generalization*
(v) *Under-generalization*
(vi) *Random response*

Error analysis or error pattern analysis is the analysis of errors in learners' mathematical workings with the aim of identifying the common patterns and to find explanations for the causes of these errors (Herholdt and Sapire, 2014).

Fiori and Zuccherri (2005) found that certain error patterns depend on different ethnic-cultural situations, teaching methods and algorithm used, and to what extent. Sri Lanka is a multi-ethnic-cultural country.

Methodology and Results

The objective of this study is to identify the patterns of errors made by the students in multiplication.

There are five categories of schools (National Schools, Type 1 AB, Type 1C, Type 2 and Type 3) in Sri Lanka. There are no elementary schools under the category National Schools. In these schools, students are taught in Sinhala, Tamil and English languages. For this study, Stratified Random Sampling Method was used to selectall schools with primary sections. Twenty four schools were chosen.

The question paper consisted of 50 multiplication sums. The Researcher designed the question paper. The question paper was administered to 980 Grades 4 and 5 students from these 24 schools. The duration to answer the questions was Two hours. All the sums (980 X 50= 49,000) were marked by the researcher. Subsequently, the patterns of errors were identified. 300 students were randomly selected out of the total sample, and interviewed to findthe reason for their answers and recorded by the researcher.

Only 185 students answered all the questions correctly. The sample answered 35,238 sums out of the 49,000 sums correctly. Therefore 13,962 (28.95%) answers were wrong. These answers were analyzed by the researcher. .

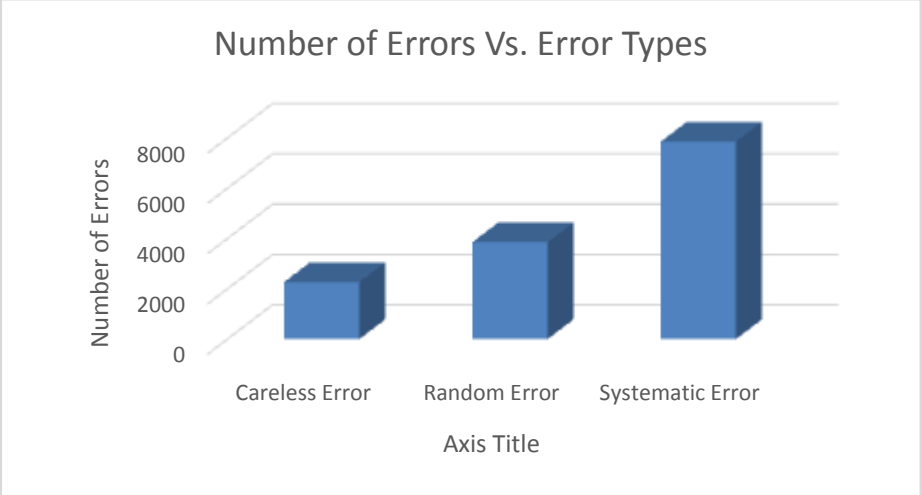
According to Cox *(1975) children's errors were classified as Random Errors, Careless Errors and Systematic Errors. The type of errors and the percentages are given in the Table below:*

Table 1

Error Type and number of wrong answers

Error Type	Number	Percentage
Careless Error	2263	16.21
Random Error	3851	27.58
Systematic Error	7848	56.21
Total	13962	100.00

Above Data can be shown in a graph as follows:



The Systematic Errors (7848) were further grouped. Yetkin (2003) reported that majority of the students' errors are often systematic and rule-based rather than non-systematic errors. This Error Analysis focusses on the Systematic Errors,which occurred consistently and prevalent in students' work that reflects lack of knowledge and skills.

Analyzing the Systematic Errors

Educators typically analyze students' mathematical errors with the intention to improve instruction and correct misconceptions (Mastropieri & Scruggs, 2002). According to Nanayakkara (1992) Systematic Errors could have arisen due to many reasons. Data collected during interviews enabled to find the most important reason. Observation of the children's answers and the interview with them enabled the researcher to identify the following ten error types.

1. Error Type I

Recalling errors: incorrectly remembering multiplication facts.

Examples

Table 2: Examples for Error Type I

E1	E2	E3	E4	E5
8 × 2 =14	7×3=20	3×4 =10	4×5 = 22	6×6 =34

2. Error Type II

Operation sign related errors: Interpret × as +

Examples

Table 3: Examples for Error Type II

E 6	E 7	E 8	E 9	E10
8 × 2 =10	7×3=10	3×4 =7	4×5 = 9	6×6 =12

3. Error Type III

Operation sign related errors: Interpret × as -

Table 4: Examples for Error Type III

E11	E12	E13	E14	E15
6 × 2= 6	10 × 2=8	9 × 3=6	15 × 2 …… 3	27 × 3 …….. 4

4. Error Type IV

Operation sign related errors: Interpret × as ÷

Table 5: Examples for Error Type IV

E16	E17	E18	E19	E20
6 × = 3	10 × 2= 5	9 × 3= 3	8 × 4= 2	10 × 5= 2

5. Error Type V

Zero related error: ignoring the function of zero as a place holder
Examples

Table 6: Examples for Error Type VI

E26	E27	E28	E29	E30
210 × 2 = 42	103 × 3 = 39	204 ×4 = 92	308 × 5 = 190	250 × 5 = 125

6. Error Type VI

Direction of operation errors: Starts multiplying from the left
Examples

Table 7: Examples for Error Type VII

E31	E32	E33	E34	E35
62 × 2 = 25	75 × 2 = 45	44 × 3 = 213	31 × 4 = 25	52 × 5 = 512

7. Error Type VII

Answer in reverse order: Partial of one's column written in hundreds column and so on.

Table 8: Examples for Error Type VII

E36	E37	E38	E39	E40
62 × 2 = 412	74 × 2 = 814	42× 3 = 612	31 × 4 = 412	51 × 5 = 525

8. Error Type VIII

Carried digit related: Carrying the incorrect digit

Table 9: Examples for Error Type VIII

E41	E42	E43	E44	E45
16 × 2 …… 3 1	45 × 3 …… 17 1	27 × 3 …… 7 2	75 × 4 = 282	89 × 5 =454

9. Error Type IX

Carried digit related: adding the carrying digit and multiply in the tenth column

Table 10: Examples for Error Type IX

E46	E47	E48	E49	E50
16 × 2 …… 4 2	45 × 3 …… 155	27 × 3 …… 121	75 × 4 = 360	89 × 5 =605

10. Error Type X

Carried digit related: Carried the digit from the ones column and not added to the partial product of tens column, but added to hundreds column

Table 11: Examples for Error Type X

E51	E52	E53	E54	E55
116 × 2 …… 3 22	125 × 3 …… 465	227 × 3 …… 861	215 × 4 = 1040	209 × 5 =1405

11. Place value related errors: Failure to understand the position of digit determine value.

Table 12:Examples for Error Type XI

E56	E57	E58	E59	E60
46 × 32 …… 92 138 …… 230	25 × 41 …… 25 100 …… 125	27 × 36 ….. 162 81 …… 243	50 × 23 ……. 150 100 ….. 250	75 ×35 ….. 375 225 …….. 600

12. Place value related errors: over generalized the rule 'put down a zero' for multiplication by the ten digit.

Table 13: **Examples for Error Type XII**

E61	E62	E63	E64	E65
46 × 32 …… 920 1380 …… 2300	25 × 41 …… 250 1000 …… 1250	27 × 36 ….. 1620 810 …… 2430	50 × 23 ……. 150 100 ….. 250	75 ×35 ….. 3750 2250 …….. 6000

The researcher grouped the children's systematic errors (7848) according to the error types

Table 14 : Error Types and number of wrong answers

Error pattern	**Error Type**	**Number**	**Percentage**
Recalling Error	Error Type I	731	9.31
Operation sign related errors	Error Type II	721	9.19
	Error Type III	412	5.25
	Error Type IV	253	3.22
Zero related	Error Type V	923	11.76
Direction of operation errors	Error Type VI	978	12.46
Answer in reverse order	Error Type VII	421	5.36
Carried digit related	Error Type VIII	592	7.54
	Error Type IX	734	9.35
	Error Type X	874	11.14
Place value related errors	Error Type XI	785	10.00
	Error Type XII	424	5.40
Total		**7848**	**100.00**

The above Table 13 shows the error-patterns and the types along with the percentage of students in the sample who made these errors. The Error Patterns consisted of seven Types *viz.* Recalling Error, Operation sign related error, Zero related Error, Direction of Operation Error, answer in reserve order, carried digit related and carried digits related error.

Significantly 28.03% % of the errors were related to carry digit related. Error Types VIII, IX and X are under this type. 7.54% of the errors were related to Error Type VIII. That is carrying incorrect digit. Example: 16 × 2 = 31. Child multiplied the unit place digit 6 × 2

and got the answer 12. Then the child wrote the digit 1 at the unit place and carried the digit 2 to tenth place. Then the child multiplied 1 × 2 and got the answer 2. Child add the 2 and carrying digit 2 (= 4). Then child written the answer as 16 × 2 = 31. The child did not understand the carried the digits.

9.35% of the errors were related to Error Type IX. Example: 16 × 2 = 42. Child first adds the carried number and then continues multiplying. For example, Child multiplies six by two to give twelve. Child then carries the one and adds itto one to give two. Next child multiplies two by two to give four.

During the interview with a Grade 5 child the researcher found that, Kumara can efficiently and correctly solve single digit problems. His errors are due to a mistake in the process it uses. This is often apparent when students are simply rote-taught a set method, but lack the mathematical understanding behind the process. Kumara needs to first realize that hismethod is faulty. This could be done through use of concrete materials. Kumara secondly needs to revise multiplying by a single digit with carrying. He needs to write the algorithm in its extended form (as shown below) until the procedure is familiar and the concepts (especially the distributive property) are well understood.

11.4% of the errors were related to Error Type X. Carried the digit from the ones column not added to the partial product of tens column, but added to hundreds column. In Example 116 × 2,tThe child multiplied the unit's place digit 6 × 2 and got the answer 12. Then the child wrote the digit 2 in the unit's place. Then it multiplied 1 × 2 = 2 and without adding the carried digit one with this 2 wrote 2 only. But the child multiplied the hundred's place digit 1 × 2 = 2 and then added the digit one which was carried from the unit place and wrote 3 in the hundred's place. This child do not have the idea of place value.

Significantly 17.66% % of the errors were related to operation sign. Error types II, III and IV are included in this operation sign related errors. 9.19 % of the errors were Error Type II. Children confused the signs × and +. Children who have the Mathematics learning disability called 'dyscalculia' made this type of errors frequently. 5.25% of the errors were Error Type III. Children confused the signs × and -. 3.22 % of the errors were Error Type III. Children confused the signs × and ÷.Children first learned the + sign then they learn - , × and ÷.

Significantly 15.4% errors related to place value errors. Error Type XI and XIIwere included in this type.10% of the errors were under the Error Type XI. Example 46 × 32

46 ×32

…….

92 ----- Step I
138 ----- Step II
……..
230

The child multiplied 46 × 2 and wrote 92 (Step I). Then the child multiplied 46 × 3 = 138(Step II). But the child did not consider the place value and wrote 138 directly. The child wrote the digit 8 of the number 138 just bottom of the 2 of the number 92. This is place value related error.

5.4% errors were related to error type XII. Example 46 × 32

46
×32
…….
920 ----- Step I
1380 ----- Step II
……..
2300

The child multiplied 46 × 2 and wrote 920 (Step I) and then multiplied 46 × 2 and wrote 92 (Step I). Then the child multiplied 46 × 3 and wrote 1380. This is over generalization error.

11.76 % errors were zero related (Error Type V). Example 210 × 2 = 42. The child did not considered the significance of 0 as a numberat this point.

12.46% of the errors were related to direction of operation (Error Type VI). Example:62 × 2 = 25. The child started to multiply from the left side. 6 × 2 = 12 then the child wrote 2 and multiply 2 × 2 and add one which carried for from the left side multiplication. Then the answer 25.

9.1 % errors were related to recalling (Error Type I). Children did not memorize the multiplication table is the main reason for this type of error.

5.36% of the errors related to reverse order. Example 62 × 2 = 412. Child first multiplied 2 × 2 and wrote 4. Then the child multiplied 6 × 2 and wrote 12 after the 4.

Conclusion

Present study highlighted that, most of the errors made by the children related to multiplication were systematic errors. Within the systematic errors incomplete operations errors are the significant errors. Systematic errors, if not arrested at the beginning, may become a serious problem to the learner later in the life as the four major mathematical operations are essential in the day to day life of a person.

The Primary Mathematics Textbooks and the Teachers' Guides give clear instructions to the primary grade teachers on the use of day to day life situations in teaching mathematics in Sri Lanka. For example, considering Division in a person's life, it is linked with almost every action of the person.In the context of the children's school life, many children do transactions by way of buying sweets or short eats at the school canteen or tuck shop. Here they have to do calculations, especially dividing, when paying for what they have bought.

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